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NONLINEAR INTERACTION OF MICROWAVE ELECTROMAGNETIC WAVES GUIDED BY THE ANISOTROPIC COMPOSITE CYLINDER

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A study is made of the linear stage of the parametric instability of electromagnetic surface waves guided by bianisotropic composite cylinder surrounded by uniform dielectric space. A three-wave interaction between an external time-harmonic magnetic field and the guided waves is investigated. The expression of instability increment of the surface waves propagating in the opposite directions is obtained. For the some practically interesting cases numerical results are discussed.

Key words: composite media, parametric instability, nonlinear interaction, instability increment.

Introduction

Electromagnetic wave propagation in composite media has been studied extensively for a long time because of many applications. In particular, the anisotropic composite material can be used in the antenna techniques for manufacturing the structural elements, microwave waveguides [1, 2] and so on. As it is known the boundary between metal and bianisotropic composite medium is able to support the surface waves. The instability of such waves has been analyzed in [3]. Propagation and excitation of electromagnetic waves in cylindrical plasma structures aligned with an external dc magnetic field have received much careful study [4]. Radiation of electromagnetic waves by given sources in a cylindrically stratified gyrotropic medium described by permittivity tensor $\hat{\varepsilon}$ and permeability tensor $\hat{\mu}$ has been considered in [5]. The parametric instability of whistler waves guided by an axially magnetized plasma column in a dielectric space has been analyzed in [6].

Here we consider the linear stage of parametric instability of electromagnetic waves guided by the bianisotropic composite cylinder surrounded by uniform dielectric space. The composite material is described by the permittivity tensor $\hat{\varepsilon}$ and permeability tensor $\hat{\mu}$ with nonzero off-diagonal elements. The axis of a cylinder is parallel to the gyrotropic axis of the medium. We analyzed the dispersion characteristics of the waves guided by a cylinder. It has been shown that bianisotropic cylinder is able to support surface waves if one of diagonal elements of any tensors $\hat{\varepsilon}$ or $\hat{\mu}$ (as well as both of them) are negative. A three wave interaction can occur if the space-time condition between the external electromagnetic field and the guided waves is fulfilled [3]. The expression of the instability increment of guided surface waves propagating in opposite directions is obtained.

Basic formulations

We take cylinder coordinate system (ρ, φ, z) with z axis is parallel to the medium gyrotropic axis. The axis of composite cylinder with radius a is parallel to the z axis. We consider the anisotropic medium which is described by the permittivity tensor $\hat{\varepsilon}$ and permeability tensor $\hat{\mu}$ in the following form

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}, \qquad \hat{\mu} = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}, \tag{1}$$

where $\varepsilon_2 = \chi\sqrt{\frac{\varepsilon_1}{\mu_1}}$, $\mu_2 = -\chi\sqrt{\frac{\mu_1}{\varepsilon_1}}$, χ is known parameter, which depends on the medium properties and can be measure [1, 2]. The cylinder is surrounded by the uniform dielectric space with permittivity ε.

We analyze the linear stage of parametric instability of waves, which appears in the presence of time – harmonic magnetic field at the frequency 2ω, which can be written in the region $0 < \rho < a$ as

$$\vec{H} = \vec{z}_0 H_0 \exp(i2\omega t).$$

It can be shown that there are two types of axisymmetric surface waves are guided by the cylinder. They are *H*-waves (with the components $H_\varphi, H_\rho, E_z, E_\rho$), and *E*-waves (with the components $E_\varphi, E_\rho, H_z, H_\rho$). The field components of the *H*- waves in the linear approximation (when the external magnetic field is absent, i.e. $H_0 = 0$) can be written in the form:

$$\rho \le a$$

$$H_\varphi^{(1,2)} = C^{(1,2)} \frac{K_1(s_H a)}{J_1(\beta_H a)} J_1(\beta_H \rho) \exp i(\omega t \mp h_H z)$$

$$H_\rho^{(1,2)} = \pm \frac{i\chi}{\sqrt{\mu_1 \varepsilon_1}} C^{(1,2)} \frac{K_1(s_H a)}{J_1(\beta_H a)} J_1(\beta_H \rho) \exp i(\omega t \mp h_H z)$$

$$E_\rho^{(1,2)} = \pm \frac{h_H}{k_0 \varepsilon_1} C^{(1,2)} \frac{K_1(s_H a)}{J_1(\beta_H a)} J_1(\beta_H \rho) \exp i(\omega t \mp h_H z)$$

$$E_z^{(1,2)} = \frac{-i\beta_H}{k_0 \varepsilon_3} C^{(1,2)} \frac{K_1(s_H a)}{J_1(\beta_H a)} J_0(\beta_H \rho) \exp i(\omega t \mp h_H z)$$

$$\rho \ge a$$

$$H_\varphi^{(1,2)} = C^{(1,2)} K_1(s_H \rho) \exp i(\omega t \mp h_H z)$$

$$E_\rho^{(1,2)} = \pm \frac{h_H}{k_0 \varepsilon} C^{(1,2)} K_1(s_H \rho) \exp i(\omega t \mp h_H z)$$

$$E_z^{(1,2)} = \frac{i s_H}{k_0 \varepsilon} C^{(1,2)} K_0(s_H \rho) \exp i(\omega t \mp h_H z),$$

here $J_m(\xi)$ is the mth-order Bessel function of the first kind, $K_m(\xi)$ is the mth-order modified Bessel function of the second kind, $k_0 = \omega\sqrt{\varepsilon_0 \mu_0}$ is the wave number of free space, β_H and s_H are transverse wave numbers referring to the composite and dielectric medium respectively

$$\beta_H^2 = \frac{\varepsilon_3}{\varepsilon_1}(k_0^2(\varepsilon_1 \mu_1 - \chi^2) - h_H{}^2), \quad s_H^2 = h_H{}^2 - k_0^2 \varepsilon.$$

The longitudinal propagation constant h_H of *H*-waves can be obtained from the dispersion equation

$$s_H \varepsilon_3 J_1(\beta_H a) K_0(s_H a) + \beta_H \varepsilon J_0(\beta_H a) K_1(s_H a) = 0. \tag{2}$$

The field components of the *E*- waves in the linear approximation can be written as

$\rho \le a$

$$E_\varphi^{(1,2)} = D^{(1,2)} \frac{K_1(s_E a)}{J_1(\beta_E a)} J_1(\beta_E \rho) \exp i(\omega t \mp h_E z)$$

$$E_\rho^{(1,2)} = \frac{\mp i\chi}{\sqrt{\mu_1\varepsilon_1}} D^{(1,2)} \frac{K_1(s_E a)}{J_1(\beta_E a)} J_1(\beta_E \rho) \exp i(\omega t \mp h_E z)$$
$$H_\rho^{(1,2)} = \frac{\mp h_E}{k_0\mu_1} D^{(1,2)} \frac{K_1(s_E a)}{J_1(\beta_E a)} J_1(\beta_E \rho) \exp i(\omega t \mp h_E z)$$
$$H_z^{(1,2)} = \frac{i\beta_E}{k_0\mu_3} D^{(1,2)} \frac{K_1(s_E a)}{J_1(\beta_E a)} J_0(\beta_E \rho) \exp i(\omega t \mp h_E z)$$
$$\rho \geq a$$
$$E_\varphi^{(1,2)} = D^{(1,2)} K_1(s_E \rho) \exp i(\omega t \mp h_E z)$$
$$H_\rho^{(1,2)} = \frac{\mp h_E}{k_0} D^{(1,2)} K_1(s_E \rho) \exp i(\omega t \mp h_E z)$$
$$H_z^{(1,2)} = \frac{-is_E}{k_0} D^{(1,2)} K_0(s_E \rho) \exp i(\omega t \mp h_E z),$$

here β_E and s_E are given by

$$\beta_E^2 = \frac{\mu_3}{\mu_1}(k_0^2(\varepsilon_1\mu_1 - \chi^2) - h_E{}^2), \qquad s_E^2 = h_E{}^2 - k_0^2\varepsilon.$$

The longitudinal propagation constant h_E of E-waves can be found from following dispersion equation

$$s_E\mu_3 I_1(\beta_E a) K_0(s_E a) + \beta_E I_0(\beta_E a) K_1(s_E a) = 0. \tag{3}$$

The investigation of the dispersion questions (2) and (3) have shown that

- in the case $\mu_3 = -\mu_1, \varepsilon_3 = -\varepsilon_1$, the composite cylinder can support *H*- and *E*- surface waves,
- in the case $\mu_3 = -\mu_1, \varepsilon_3 = \varepsilon_1$, composite cylinder is able to support *E*- surface waves.
- in the case $\mu_3 = \mu_1, \varepsilon_3 = -\varepsilon_1$, composite cylinder can guide *H*- surface waves.

Nonlinear interaction of surface waves

The nonlinear interaction of the surface waves at the frequency $\omega_1 = \omega_2 = \omega$ guided by the cylinder and propagating in the opposite directions of the *z* axis ($\vec{h}_1 = -\vec{h}_2$) may observe in the presence of the external time-harmonic magnetic field at the frequency 2ω. A three wave interaction can occur if $\omega_1 + \omega_2 = 2\omega$. To discuss the parametric instability of surface waves we start from the Maxwell's equations

$$rot\vec{H}^{(1,2)} = \varepsilon_0\hat{\varepsilon}\frac{\partial \vec{E}^{(1,2)}}{\partial t},$$
$$rot\vec{E}^{(1,2)} = -\mu_0\hat{\mu}(H_0)\frac{\partial \vec{H}^{(1,2)}}{\partial t}, \tag{4}$$

here H_0 is the amplitude of the external magnetic field. As it is known the external magnetic field may influence on the material parameters of a composite medium. For the first approximation we assume that the tensor of magnetic permeability can be written in the following form

$$\hat{\mu}(H_0) = \begin{pmatrix} \mu_1 + \alpha H_0 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 + \alpha H_0 & 0 \\ 0 & 0 & \mu_3 + \alpha H_0 \end{pmatrix}, \tag{5}$$

where α is the small parameter. Below we consider the case when the following relation takes place $|\mu_1| = |\mu_3|$.

In the approximation of a week nonlinearity the Maxwell's equations can be rewritten in the form

$$rot\vec{H}^{(1,2)} = \varepsilon_0 \hat{\varepsilon} \frac{\partial \vec{E}^{(1,2)}}{\partial t},$$

$$rot\vec{E}^{(1,2)} = -\mu_0 \hat{\mu}(0) \frac{\partial \vec{H}^{(1,2)}}{\partial t} - \mu_0 \alpha H_0 \frac{\partial \vec{H}^{(1,2)*}}{\partial t}. \qquad (6)$$

The using the standard procedure [3] we can obtain the following equations for the amplitude of the surface waves:

$$\frac{\partial A_{1,2}}{\partial z} + \frac{1}{\upsilon^g_{1,2}} \frac{\partial A_{1,2}}{\partial t} = \frac{1}{N_{1,2}} \int_0^\infty \left(-\mu_0 \alpha H_0 \frac{\partial \vec{H}^{(2,1)*}}{\partial t} \vec{H}_T^{(1,2)} \right) 2\pi\rho d\rho = \sigma_{1,2} A^*_{2,1} H_0. \qquad (7)$$

Here $N_{1,2}$ are the norm of the surface wave and $\vec{H}_T^{(1,2)}$ is the magnetic field of the surface waves in the medium described by the transposed tensors, $A_{1,2}$ is the amplitude either *E*-surface waves or *H*-surface waves, $\upsilon^g_{1,2}$ is the group velocity of the waves, $\sigma_{1,2}$ are the coefficients of the nonlinear interaction.

The norm of the *H*-surface waves has the form

$$N_H^{1,2} = \pm 2\pi \frac{h_H a^2}{k_0} \{ K_1^2(s_H a) \left(\frac{1}{\varepsilon_1} - \frac{1}{\varepsilon} \right) -$$

$$- \frac{2K_1^2(s_H a)}{a} \left(\frac{1}{\varepsilon_1 \beta_H} \frac{J_0(\beta_H a)}{J_1(\beta_H a)} - \frac{1}{\varepsilon s_H} \frac{K_0(s_H a)}{K_1(s_H a)} \right) +$$

$$+ K_1^2(s_H a) \left(\frac{1}{\varepsilon_1} \frac{J_0^2(\beta_H a)}{J_1^2(\beta_H a)} - \frac{1}{\varepsilon} \frac{K_0^2(s_H a)}{K_1^2(s_H a)} \right) \}.$$

The norm of the *E*-surface waves is given by

$$N_E^{1,2} = \mp 2\pi \frac{h_E a^2}{k_0} \{ K_1^2(s_E a) \left(\frac{1}{\mu_1} - 1 \right) -$$

$$- \frac{2K_1^2(s_E a)}{a} \left(\frac{1}{\mu_1 \beta_E} \frac{J_0(\beta_E a)}{J_1(\beta_E a)} - \frac{1}{s_E} \frac{K_0(s_E a)}{K_1(s_E a)} \right) +$$

$$+ K_1^2(s_E a) \left(\frac{1}{\mu_1} \frac{J_0^2(\beta_E a)}{J_1^2(\beta_E a)} - \frac{K_0^2(s_E a)}{K_1^2(s_E a)} \right) \}.$$

The group velocity can be written as

$$\left(\upsilon^g_{1,2} \right)_{H,E} = \pm c \frac{h_{H,E}}{k_0} V_{H,E},$$

here functions $V^{H,E}$ are

$$V_H = \varepsilon^{-1} \{ \varepsilon_1 \beta_H J_1(\beta_H a) K_0(s_H a) - \varepsilon s_H J_0(\beta_H a) K_1(s_H a) \} \times$$

$$\times \{ \varepsilon_1 \beta_H J_1(\beta_H a) K_0(s_H a) - s_H (\mu_1 \varepsilon_1 - \chi^2) J_0(\beta_H a) K_1(s_H a) \}^{-1},$$

$$V_E = \{ \mu_1 \beta_E J_1(\beta_E a) K_0(s_E a) - s_E J_0(\beta_E a) K_1(s_E a) \} \times$$

$$\times \{ \mu_1 \beta_E J_1(\beta_E a) K_0(s_E a) - s_E (\mu_1 \varepsilon_1 - \chi^2) J_0(\beta_E a) K_1(s_E a) \}^{-1}.$$

The coefficients of the nonlinear interaction $\sigma_{1,2}$ can be expressed as

$$\sigma_H^{1,2} = \frac{i\pi\alpha\mu_0\omega a^2}{N_H^{1,2}}(1+\frac{\chi^2}{\mu_1\varepsilon_1})\frac{K_1^2(s_H a)}{J_1^2(\beta_H a)}\left[J_1^2(\beta_H a)+J_0^2(\beta_H a)-\frac{2}{\beta_H a}J_0(\beta_H a)J_1(\beta_H a)\right],$$

$$\sigma_E^{1,2} = \frac{i\pi\alpha\mu_0\omega a}{k_0{}^2 N_H^{1,2}}\frac{K_1^2(s_E a)}{J_1^2(\beta_E a)}\left[\left(\frac{h_E^2}{\mu_1^2}+\frac{\beta_E^2}{\mu_3^2}\right)\left(J_1^2(\beta_E a)+J_0^2(\beta_E a)\right)-\frac{2h_E^2}{\mu_1^2\beta_E a}J_0(\beta_E a)J_1(\beta_E a)\right].$$

For the homogeneous case ($\frac{\partial}{\partial z}=0$) the expression of instability increment of the surface waves of *H*- and *E*- polarization is given in the form

$$\gamma_{1,2} = \sqrt{\sigma_{1,2}\sigma_{2,1}^{*}\upsilon_{1,2}^{g}\upsilon_{2,1}^{g}}\,|H_0|.$$

The parametric instability is observed if the following condition takes please $\sigma_{1,2}\sigma_{2,1}^{*}\upsilon_{1,2}^{g}\upsilon_{2,1}^{g}>0$.

Numerical results and discussion

We now present the numerical results for the instability increments of the surface waves performed for medium parameters given in [1]: $\mu_1=5,\ \varepsilon_1=1,\ \chi=0.1$. The numerical calculations performed for the waves with the frequency $\omega=2\pi\cdot10^9\,\mathrm{s}^{-1}$. For this set of parameters and for the radius of cylinder $a=0.03\,\mathrm{m}$ the values of longitudinal propagation constant *h* and transverse wave numbers β of the surface mode with the least magnitude of β are: $h_H=3.12k_0,\ s_H=2.96k_0,\ \beta_H=2.18k_0$ (if $\mu_3=\pm\mu_1$ and $\varepsilon_3=-\varepsilon_1$); $h_E=6.8k_0,\ s_E=6.74k_0,$ $\beta_E=6.44k_0$ (if $\mu_3=-\mu_1$ and $\varepsilon_3=\pm\varepsilon_1$).

Numerical calculation has shown, that for chosen set of parameters, the condition $\sigma_{1,2}\sigma_{2,1}^{*}\upsilon_{1,2}^{g}\upsilon_{2,1}^{g}>0$ takes please, and the parametric instability of surface waves can be observed.

The numerical results of normalized instability increment $\tilde\gamma=\frac{\gamma}{\alpha H_0\omega}$ of the surface waves as function of mode number *n* are presented in Figs. 1, 2. We number the modes of a cylinder in the order of an increase of their transverse wave number β starting with the minimum value of β and using the numbers *n*=1,2,3…. Normalized instability increment $\tilde\gamma$ is presented for the anisotropic cylinder with radius $a=0.03\mathrm{m}$. Figs.1, 2 show the instability increment of axisymmetric *E*- and *H*-surface waves with different number of modes *n* respectively.

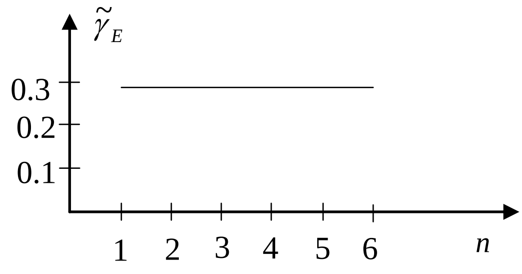

Fig. 1. The instability increment of the *E* - surface waves versus *n*, $a=0.03\mathrm{m}$

One can observe from Fig. 1, 2 that an increment $\tilde\gamma$ weekly depends on the mode number.

Fig. 3 shows the instability increment $\tilde\gamma_E$ of the first mode of surface waves as a function of cylinder radius. The behavior of the instability increment $\tilde\gamma_H$ is similar to $\tilde\gamma_E$ and we do not give it here.

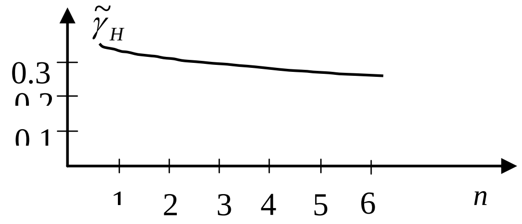

Fig. 2 The instability increment of the *H* - surface waves versus *n*, $a = 0.03$m

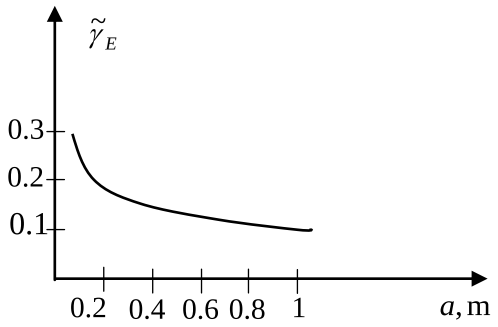

Fig. 3. The instability increment $\tilde{\gamma}_E$ of the surface waves with *n*=1 as versus cylinder radius *a*

One can see that increment $\tilde{\gamma}_E$ decreases rather rapidly with an increasing the cylinder radius.

Conclusion

Thus, we consider the parametric instability of the surface waves guided by the composite cylinder in the presence of time-harmonic external magnetic field. The instability growth rate is analyzed. The results obtained can be useful for creating microwave devices made of composite material.

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Subject: The study of parametric instability of electromagnetic surface waves guided by the composite cylinder surrounded by uniform dielectric space is made. The composite medium inside the cylinder is described by permittivity and permeability tensors with nonzero off-diagonal elements.
Method/approach: The dispersion characteristics of waves supported by the composite cylinder are analyzed. It was shown that parametric instability of surface waves propagating in the opposite directions can arise in the presence of time-harmonic external magnetic field aligned with the axis of the cylinder. The instability of waves can occur if the space-time condition between the external field and guided waves is fulfilled. In the approximation of week nonlinearity the equations for the amplitudes of the surface waves are derived.
Results: We consider the parametric instability of surface waves guided by bianisotropc cylinder in the presence of external magnetic field. An expression of instability increment of guided waves has been obtained and analyzed as a function of the cylinder parameters. For the some practically interesting cases numerical analyses have been performed.
Range of application: Over the past decade, there has been shown a substantial degree of interest in composite material. The composite material can be used for manufacturing of microwave waveguides, antenna systems, the structural elements of microwave devices and so on. It should be taken into account that the instability of waves guided by the boundary of the antenna structures can influence on the operating of antenna systems.

Key words: composite media, parametric instability, nonlinear interaction, instability increment.