Design of the corrugated-core sandwich panel with external active cooling system

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\textbf{ABSTRACT}

Optimal structure of thermal barrier skins used for rescue vehicles experiencing extreme conditions is developed. The conditions include extreme arctic cold and possible extreme heat of burning oil. The skin structure includes fibrous insulation material as well as external active cooling system using sprinklers. Optimal design variables, for the best combinations of the thermal and mechanical performances – the panel geometry and the discharge density – are examined by the analytic and numerical means. It is shown that the high discharge density in the cooling system may be necessary not only for the thermal protection, but also to provide the strength of the panel elements. In particular under the considered loading conditions, the solution of the optimization problem with all constraints exists only for the enough high discharge density due to the thermal buckling of the web elements inside the panel under non-uniform heating.

\section{1. Introduction}

We examine the optimal structure of thermal barrier skins used for the rescue vehicles experiencing extreme conditions. The conditions include extreme arctic cold and possible extreme heat (up to 1200 °C) of burning oil\textsuperscript{[1]}. The skin structure includes fibrous insulation material as well as active cooling system using sprinklers. The optimal design, for the best combinations of the thermal and mechanical performances, involves the optimal choice of the panel geometry and of the discharge density.

Fiberglass is usually the structural material of choice for the similar vehicles or the freefall lifeboats; it possesses sufficient specific strength and stiffness. However, for the thermal protection against external cold and heat conditions the additional insulation is needed. As discussed in\textsuperscript{[1,2]}, the use of only the passive thermal protection leads to substantial increase of panel thickness and weight that may be unacceptable. Hence the passive protection should be supplemented by an active external cooling (sprinkler system). The coolant in this system may be seawater; however, in its absence, one may be compelled to use the onboard supply. This is the reason we consider not only the mass minimization problem, but, also, the minimization of the discharge density in the cooling system.

We consider a panel with a corrugated core. General principles of optimal design of such panels have been developed earlier (see, for example,\textsuperscript{[3–7]}). Note that sandwich panels with honeycomb core have higher mass efficiency, but their thermal protection characteristics are somewhat lower than those of corrugated core sandwiches, due to higher values of the effective thermal conductivity in the transverse direction\textsuperscript{[8,9]}. As far as foam core is concerned, it has good thermal insulation properties but poor mechanical characteristics\textsuperscript{[8–10]}.

Sandwich panels with corrugated cores are often the best option for multifunctional structures: they have sufficient load bearing capacity and thermal protection\textsuperscript{[9]}. The design of such panels has been discussed in a number of works; for the passive thermal protection, see the works\textsuperscript{[11–14]}. It has been shown, in particular, that the conditions of thermal protection and thermal buckling of the core elements constitute the most serious constraint. It has also been found that analytical one-dimensional solutions allow one to obtain sufficiently accurate estimates of the thermal state of the panels under transient heating conditions across thickness. The panels with an internal active convective cooling system that use water were analyzed, in the context of structural and hydrodynamic parameters, in\textsuperscript{[14–17]}.

The present work aims at the optimization of geometry of a load-bearing and thermal protection panel with an active external cooling...
system. The interior of the panel contains an insulating fibrous material, to provide the passive thermal protection. We note that the active cooling system may be of three distinctly different types: transpiration, film cooling, and convective cooling [18]. The sprinkler system produces a “film” cooling (a thin layer of water flow on the vehicle surface). We proposed a simplified evaluation of the thermal state of the panel with such cooling system.

We solve the optimization problem using the methodology of optimization under constraints. Finite element simulations are carried out, and compared to the analytical solution. Optimal variants of panel structure are identified.

2. Modeling of the structure of the panel

We suggest simple analytical models for the effective thermal properties of the panel, for the cooling process, for the structural strength of the panel under mechanical loading and for thermal buckling of its elements caused by non-uniform temperature distribution.

2.1. Structure of the panel and its effective thermal properties

We consider a sandwich panel with corrugated core (a “web”) shown in Fig. 1 where notations are as follows. The face thickness is \( t_f \), the web thickness is \( t_c \) and the core depth is \( h_c \). The distance between the web elements is \( d_f \), the corrugation pitch is \( 2p \), and the angle between the web and the vertical direction is \( \theta \). The total panel thickness is \( h = h_c + 2t_f \). Total area of the load bearing elements in the panel cross section is \( A = 4t_f p + 2t_c (d_f + h_c / \cos \theta) \). The panel length \( a = 1200 \text{ mm} \) and its width \( b = 500 \text{ mm} \). Heat-insulating fibrous material fills the free space inside the panel. In the following, we use parameter \( N = a / (2p) \) for the number of core pitches. Internal panel surface is located at \( z = 0 \) and the external one at \( z = h \) in the coordinate system shown in Fig. 1(b). The panel is placed on the lateral vertical wall of the vehicle that has mass \( M \), length \( L \), width is \( W \), and height \( H \).

Thus, the average mass density of the panel is

\[
\rho = \frac{2t_f V_f + \rho_c V_c + \rho_f V_f}{V} \tag{1}
\]

The effective heat capacity and thermal conductivity, evaluated by the law of mixtures (that was shown in works [1,11,12] to be sufficiently accurate for the structures of this kind) are given by

\[
c = \frac{2t_f C_f V_f + \rho_c C_c V_c + \rho_f C_f V_f}{\rho V} \tag{2}
\]

\[
k = \frac{2k_f V_f + k_c V_c + k_f V_f}{V} \tag{3}
\]

2.2. Analysis of the external cooling process

For thermal protection of the vehicle, as it passes through burning oil, an external sprinkler system is used (Fig. 2). This system supplies water (or other cooling liquid) through the sprinkler heads mounted at the top of the vehicle; its temperature will be assumed \( T_0 = 20^{\circ} \text{C} \). Water flows down along the outer surface of the vehicle under the action of gravity, thus protecting the vehicle. The discharge density \( \eta \) (that specifies how much water is spread, per minute, over a part of the cooled surface area of one square meter) is usually below \( 20 \text{ L·min}^{-1} \cdot \text{m}^{-2} \) (or \( 3.33 \times 10^{-4} \text{ m/s} \) in the Si-system).

For the analysis of the cooling process, the following assumptions will be used:

1) The flow of water is laminar, of constant thickness \( h_w \) (Fig. 2);
2) The flow is uniformly heated through the thickness to temperature \( T_w (x,y) \);
3) No boiling occurs;
4) The heat dissipation due to evaporation is neglected.

The thickness of the water flow \( h_w \) is controlled by the discharge density \( \eta \) and flow velocity \( v_w \). Water moves, driven by gravity, along
the vertical wall of the vehicle having height $H$. The flow velocity at the lower edge of the vehicle body ($y = H$) is $v_y = \sqrt{2gH}$. The discharge density $\eta$ is the ratio of the water volume flowing, per second, over a part of the vehicle surface of the area of 1 m$^2$. Then,

$$\eta = \frac{v_y h_w}{H} \quad \text{so that} \quad h_w = \eta \sqrt{\frac{H}{2g}} \quad \text{(4)}$$

To estimate the maximum temperature $T_{w,\text{max}}$ to which the water is heated during its run along the vehicle surface. We assume Newton’s law for the heat transfer between layer of cooling water and external environment and a thermal insulation condition between water and vehicle’s body. Neglecting the conductive heat transfer in the direction of flow, one obtains the following relation for a unit volume of the cooling water:

$$\alpha_w (T_w - T_h) = c_{w} \rho_w h_w \frac{\delta T}{\delta t} \quad \text{(5)}$$

where $\alpha_w$ is the coefficient of heat transfer across the boundary between the flow and the external environment, $T_h$ is the (elevated) temperature of the external environment, and $c_{w} \rho_w$ are the heat capacity of the water and its density.

Solving Eq. (5) with the initial condition $T_w = T_h$ and taking into account that the time the water spends on the vehicle’s surface is $\sqrt{H/2g}$, and also taking into account (4), we find the maximal temperature of water:

$$T_{w,\text{max}} = T_h - (T_h - T_h)e^{-\frac{H h_w}{c_{w} \rho_w \sqrt{2g}}} \quad \text{(6)}$$

We now obtain the upper and lower bounds of the discharge density that ensures the required thermal protection of the structure. To obtain the lower bound, it is assumed that the maximal temperature of the water $T_{w,\text{max}}$ should not exceed its boiling point $T_b$. Then Eq. (6) implies

$$\eta_{\text{min}} = \frac{\alpha_w}{c_{w} \rho_w \ln((T_h - T_h)/(T_h - T_h))} \quad \text{(7)}$$

The upper bound $\eta_{\text{upp}}$ follows from the condition that the temperature of the water, and therefore the temperature of the outer surface of the panel, should not exceed a given permissible value that is determined by the safety conditions (see Section 2.3 below). Then

$$\eta_{\text{max}} = \frac{\alpha_w}{c_{w} \rho_w \ln((T_h - T_h)/(T_h - T_h))} \quad \text{(8)}$$

2.3. Thermal analysis

The two basic insulating functions of the panel are: thermal insulation under cold environment conditions and thermal protection under intense heating (burning oil). In the first case, the temperature of the inner surface of the panel $T_{wi}$ should be above the prescribed $T_{wi,\text{min}}$ (safety requirement). The value $T_{wi}$ is determined from the solution of the steady-state heat conduction problem. Thus, we obtain the first optimization constraint (as discussed in detail in [1]):

$$T_{wi} = T_i + \frac{(T_i - T_i)/\alpha_i}{h / k + 1/a_i + 1/a_i} \geq T_{wi,\text{min}} \quad \text{(9)}$$

where $T_w$, $T_i$ are air temperatures outside and inside the vehicle and $\alpha_i, a_i$ are the heat transfer coefficients on the external and inner surfaces of the panel.

In the second constraint (motion in the burning oil field for the specified time period $\Delta t$) the external cooling system operates. Eq. (6) determines the maximal temperature of the coolant flowing along the panel surface – which is also the temperature on the external panel wall. To determine the temperature distribution in the panel, we need to solve the transient heat conduction problem with the condition that constant temperature $T_{x,\text{max}}$ given by (6) is prescribed on its outer surface for a period of time $\Delta t$. On the inner surface, we assume the thermal insulation condition. Thus, we consider the following problem:

$$0 < z < h: \quad \frac{\partial^2 T}{\partial z^2} = k \frac{\partial T}{\partial x}$$

$$z = 0: \quad \frac{\delta T}{\delta z} = 0, \quad z = h: \quad T = T_{x,\text{max}}, \quad t = 0: \quad T = T_0 \quad \text{(10)}$$

The solution of this problem can be found analytically [19]. The solution of the problem (10) yields the second constraint: the maximal temperature of the inner surface of the panel $z = 0$ that is realized after time period $\Delta t$ should not exceed a given permissible value:

$$T_{x,\text{max}} = T_{w,\text{max}} - 2(T_{w,\text{max}} - T_h) \sum_{n=0}^{\infty} \left(\frac{-1}{\rho_h}\right)^n e^{-h \frac{\delta z}{w} \Delta t} < T_{\text{max}} \quad \text{(11)}$$

where $\rho_h = (2n + 1)\pi/2, W = k/(\varepsilon \rho b^2)$.

Note that all the assumptions made in derivation of condition (11) provide us a reserve for the heat protection parameters. We neglected the initial velocity of the coolant to determine the flow velocity, neglected its evaporation, used the maximum temperature of coolant to assess the thermal state of the panel, and we assumed the thermal insulation conditions in solving problems (5) and (10).

2.4. Strength analysis

We discuss requirements on strength of the panel under different loading conditions – both static and dynamic – that correspond to different working regimes of the vehicle. Restrictions for the strength, stability and thermal stability of panel elements are specified.

2.4.1. Compression

The maximal compression resultant force $N_y$ (in the parallel to the panel direction $y$) corresponds to the case when the vehicle overturns and lands on its roof. Assuming, for simplicity, the uniform normal traction boundary conditions we have the same normal stress $\sigma_y = 2pN_y/A$ in both face sheets and the core. If the boundary load – and hence the stress – are sufficiently high, one of the three possible failure mechanisms can be identified: fracture of the bulk material, as induced by the compressive stress; buckling of the face sheet, and buckling of web elements. Using known results [4, 20] for the critical stress that causes buckling, we have the following critical conditions corresponding to the mentioned mechanisms:

$$\sigma_y = \sigma_{\text{fl}} \quad \text{face or web failure}$$

$$\sigma_y = \frac{\pi^2 E t_y^3}{3(1 - \nu^2)(2p - d_j)^2} \quad \text{face sheet buckling}$$

$$\sigma_y = \frac{\pi^2 E t_y^3 \cos^2 \theta}{3(1 - \nu^2)h_i^2} \quad \text{web element buckling}$$

where $\sigma_{\text{fl}}$ is compressive strength of the panel material.

2.4.2. Shear

In the case of asymmetric motion over an obstacle, the vehicle body is twisted and resultant shear force $N_{xy}$ occurs in the panel. This force is primarily resisted by the panel faces. Shear stress in the faces is $\tau_{xy} = N_{xy}/(2t_y)$. The critical stresses under shear are [4, 20]:

$$\tau_{xy} = \tau_{\text{fl}} \quad \text{face or web failure}$$

$$\tau_{xy} = \frac{\pi^2 \sqrt{3} E t_y^3}{3(1 - \nu^2)(2p - d_j)^2} \quad \text{face sheet buckling}$$

$$\tau_{xy} = \frac{\pi^2 \sqrt{3} E t_y^3 \cos^2 \theta}{3(1 - \nu^2)h_i^2} \quad \text{web element buckling}$$

where $\tau_{\text{fl}}$ is shear strength of the panel material.

2.4.3. Impact

In a preliminary design, we assume that, under impact with flat
obstacle, the uniform pressure $q$ acts on the lateral surface of the vehicle with area $L \times H$. The compression of the panel in the transverse direction is primarily resisted by the web elements. The mean relative displacement between the outer and the inner faces of the panel is $\delta$. This displacement determines the work of external forces done under impact. For the given initial vehicle velocity $v_0$, pressure $q$ can be estimated from the energy consideration as $q = \frac{M_0^2}{2LH}$. Displacement $\delta$ can be evaluated as $\delta = \varepsilon_{\text{max}} h \cos \theta$ if the maximum strain in the web elements $\varepsilon_{\text{max}}$ is known. Thus, we find the value of pressure:

$$
q = \frac{M_0^2}{2LH} = \frac{M_0^2}{2\varepsilon_{\text{max}} h L H \cos \theta}
$$

Compression normal stress in the web elements $\sigma_n$ can be estimated from the equilibrium equation in the $x$ direction:

$$
\sigma_n = \frac{p q}{t \cos \theta}
$$

(15)

The critical normal stress for the web element under compression is [21]:

$$
\sigma_n \geq 2\pi^2 E \left( \frac{L}{1-\nu^2} \right)^2 (t \cos \theta)^2
$$

(16)

Using Eqs. (14)(16) and assuming that maximum strain in the web elements under impact loading is proportional to its critical buckling strain under static loading i.e. $\varepsilon_{\text{max}} = n \varepsilon_p = n \varepsilon_p (1-\nu^2)/E$, we could derive the condition of web element stability under impact in the form:

$$
8 n \pi^4 E t \cos \theta \nu \frac{\varepsilon_p}{p} > \frac{M_0^2}{L H}
$$

(17)

The coefficient $n$ can be determined based on comparison of (17) with FE transient structural analysis with large deflection effects. As it will be shown below, the optimal value of this coefficient is $n = 32$.

2.4.4. Thermal buckling

Due to non-uniform heating of the panel, thermal buckling of web elements inside it may occur, reducing the load-bearing capacity of the panel. To prevent such effects, we introduce an additional thermal buckling condition in the thermo-structural optimization problem based on the known analytical solution [22]. In this solution, a rectangular simply-supported plate is considered. Long edges of the plate are fixed in the normal direction. Temperature variation is realized along the short edges. Using this model for the web elements which short edges height is $h \cos \theta$, we find that thermal buckling occurs if their average temperature $T$ reaches a critical value [22]:

$$
T_{\text{cr}} = T_0 + \frac{\pi^4 E (1-\nu^2) h^2}{8(1-\nu^2) h^2 \alpha}
$$

(18)

where $T_0$ is the initial temperature of the panel provided there are no thermal stresses and $\alpha$ is the coefficient of thermal expansion.

Note that condition (18) includes an average temperature of the web elements only; it does not take into account the temperature distribution. The use of such estimate is possible if the temperature gradients are not overly large as is the case in the considered problem. We also note that the condition (18) assumes that the thickness of the panel can freely increase due to heating, but its in-plane displacements are equals zero, due to the fact that the panel is placed on a rigid frame, which is heated up more slowly.

2.5. Optimization

The goal is to find the geometric parameters $t_f$, $t_c$, $h_c$, $d_f$, and $N$ that minimize the mass per unit area of the panel. We also need to determine the discharge density required in the external cooling system: it should be as small as possible, as the amount of available water may be limited. Mass per unit area of the panel is calculated, taking into account (1) as $m = \rho h$.

The value of discharge $q$ is determined from (11) taking into account (6), or directly using bounds (7), (8) in which the discharge is determined only through the given physical properties of water and the parameters of heat exchange with the external environment.

The constraints in the optimization problem are formulated using safety factors. Based on (9), (11), (12), (13), (17) and (18) we introduce the following safety factors:

$$
K_{T,\text{min}} = \frac{T_f - T_0}{h / K + 1 / \alpha + 1 / \eta} \quad \text{insulation under cooling}
$$

$$
K_{T,\text{max}} = \frac{T_{\text{ maxi}} - 2(T_{\text{ maxi}} - T_{\text{ mini}})}{P_c}
$$

(19)

where

$$
K_{\text{f, fail}} = \frac{\sigma_{\text{crit}}}{\sigma_f}
$$

(20)

face buckling under compression

$$
K_{\text{b, fail}} = \frac{\pi^2 E t_f^2}{3\eta (1-\nu^2)(2p-d_i)^2}
$$

(21)

web buckling under compression

$$
K_{\text{y, shear}} = \frac{\tau_{\text{crit}}}{\tau_{\text{f, shear}}}
$$

(22)

face buckling under shear

$$
K_{\text{y, b, fail}} = \frac{\pi^2 E t_f^2 \cos \theta \nu}{3 \sin \theta (1-\nu^2) h_c \tau_{\text{f, b, fail}}}
$$

(23)

web buckling under shear

$$
K_{\text{h, b, fail}} = \frac{4\pi^4 E t_f^2 \cos \theta \nu}{(1-\nu^2) h_c^2 \alpha} (T - T_0)
$$

(24)
usual requirements for these vehicles. Material of the panel faces and core is glass fiber reinforced plastic (GFRP). All laminates assumed to be quasi-isotropic and symmetric. In the internal free space of the panel, a rockwool fibrous thermal insulation material is placed. Water is used in the external cooling system. Material properties used in the estimations are presented in Table 1. The ranges of design variables are presented in Table 2; that take into account technological limitations.

The main conditions for the temperature and mechanical loading of the panel have been formulated in the earlier work [1,2]. The minimal temperature of the external environment in formula (9) should be taken as \( T_e = -50\,^\circ C \); the temperature inside the vehicle is \( T_f = 20\,^\circ C \). The minimal allowable temperature of the panel inner surface is \( T_{\text{in}} = 12\,^\circ C \). The heat transfer coefficients in (9) are assumed to be \( a_i = 5\, \text{W m}^{-2}\,\text{K}^{-1} \) (free convection) and \( a_d = 20\, \text{W m}^{-2}\,\text{K}^{-1} \) (forced convection under conditions of strong wind and high humidity).

The maximal allowable time of the vehicle moving in the burning oil field (the outside temperature \( T_e = 1200\,^\circ C \)) is \( \Delta t = 8\,\text{min} \). The coolant temperature is \( T_f = 20\,^\circ C \), and the maximal allowable temperature of the inner surface of the panel is \( T_{\text{max}} = 50\,^\circ C \). The theory of heat transfer [23] implies that the heat transfer coefficient between the cooling water flow and external environment at given elevated temperature should be about \( \alpha_w = 20\, \text{W m}^{-2}\,\text{K}^{-1} \).

On the basis of (7) and (8), we obtain the upper and lower bounds of the discharge density of the cooling system. Given that the coolant is water, which should not overheat above \( T_{\text{u,max}} = 90\,^\circ C \), we find the lower bound \( \eta_{\text{min}} = 4.7\,\text{L min}^{-1}\,\text{m}^{-2} \). The upper bound follows from the condition \( T_{\text{u,max}} = T_{\text{max}} = 50\,^\circ C \), and therefore \( \eta_{\text{max}} = 11.1\,\text{L min}^{-1}\,\text{m}^{-2} \).

The maximal compression force per unit of the panel length that occurs due to overturning of the vehicle can be estimated as: \( N_r = M g/(2(L + W)) \approx 5400\,\text{N/m}. \) The maximal torque in the vehicle caused by asymmetric driving over an obstacle is \( M \approx 8800\,\text{N m} \) (as determined by FE calculations for similar metal structures, from the typical condition that their maximal twisting angle is \( \theta^\circ \)). The shear force per unit of the panel length determined from the solution of the strength of materials problem for the thin-walled section (vehicle body) torsion is: \( N_r = M/(2H) \approx 600\,\text{N/m} \). To estimate the load during impact, the initial vehicle speed, at the moment of impact, is assumed to be \( v = 2\,\text{m/s} \).

### Table 1

Materials properties.

<table>
<thead>
<tr>
<th>Material</th>
<th>( \rho ,\text{kg/m}^3 )</th>
<th>( k ,\text{W m}^{-1},\text{C}^{-1} )</th>
<th>( c ,\text{J kg}^{-1},\text{C}^{-1} )</th>
<th>( a \times 10^{-4},\text{C}^{-1} )</th>
<th>( E ,\text{GPa} )</th>
<th>( \nu )</th>
<th>( \sigma_{\text{fl}} ,\text{MPa} )</th>
<th>( \tau_{\text{fl}} ,\text{MPa} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>GFRP</td>
<td>1800</td>
<td>1</td>
<td>1000</td>
<td>25</td>
<td>22</td>
<td>0.25</td>
<td>380</td>
<td>45</td>
</tr>
<tr>
<td>Rockwool</td>
<td>25</td>
<td>0.03</td>
<td>1000</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Water</td>
<td>1000</td>
<td>0.6</td>
<td>4200</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

\( * \) Web core is triangular if \( d_f = 0 \) and rectangular if \( d_f = p = a/(2N) \).

### Table 2

Ranges of the design variables.

<table>
<thead>
<tr>
<th>Parameter ( t_i )</th>
<th>Minimum value</th>
<th>Maximum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_i ) ( \text{mm} )</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>( t_f ) ( \text{mm} )</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>( h_i ) ( \text{mm} )</td>
<td>20</td>
<td>200</td>
</tr>
<tr>
<td>( d_f ) ( \text{mm} )</td>
<td>0</td>
<td>( p )</td>
</tr>
<tr>
<td>( N )</td>
<td>–</td>
<td>10</td>
</tr>
</tbody>
</table>

Fig. 3. Geometry of the unit cell with optimal thermal (a), mechanical (b) and thermomechanical (c) performance.

### 3.1. Optimal thermal performance

At first, we find the geometry of the panels that ensures optimal thermal performance under cooling and heating conditions. We consider two limiting cases, for the minimal and maximal water discharge density (7), (8), that is necessary under conditions of intense heating. Thus, the optimal panel configuration corresponding to minimum mass is found from solution of the following problem:

\[
\begin{align*}
& m(t_i, t_f, h, d, N) \to \min, \\
& \eta = \eta_{\text{min}} \text{ or } \eta = \eta_{\text{max}}, \quad K_r_{\text{min}} > 0, \quad K_r_{\text{max}} > 0
\end{align*}
\]

Solution of this problem is the geometry of the panel with minimal thickness of the load bearing elements and with triangular core with maximal angle of inclination allowable for the considered total width of the panel (see Fig. 3(a) and Table 3). The only essential parameter in the optimization problem (19) is core depth \( h_c \), and the other design parameters could be set to its minimum values in the predefined ranges. The panel mass per unit area is 7.4 kg/m^2 and the total thickness is 91 mm.

In problem (19), the condition for thermal insulation at low temperatures \( K_{r_{\text{min}}} > 0 \) is most important; it determines the value of core depth \( h_c \). The good consistency of analytical and numerical predictions with respect to this condition has been demonstrated in the earlier work [1]. Condition at elevated temperature \( K_{r_{\text{max}}} > 0 \) is satisfied in the problem (19) with a large margin, which means that it is sufficient to use the minimum water flow \( \eta_{\text{min}} \) in the cooling system in the calculations. However, it is important to check this result using FE calculations. This is necessary, since the analytical solution does not take into account the fact that after leaving of the vehicle from a burning oil field, the temperature of internal surface of the panel can continue to rise because of the spreading of heat accumulated inside its core [1].

In FE simulations, we assume that, on the outer surface of the panel during the above-mentioned 8 min period, temperature is
Then the structure cools down under conditions of free convection, at ambient temperature of $T_0 = 20^\circ$C. On the outer surface of the panel in this case we set the heat transfer condition according to Newton's law, with the heat transfer coefficient $5 \text{ W m}^{-2} \text{ K}^{-1}$. On the inner surface of the panel we use the thermal insulation condition. The results of the calculations are shown in Fig. 4. It is shown here that the analytical solution yields a sufficient thickness of the panel for a given minimum discharge $\eta_{\text{min}}$ that provides heat protection of the internal space of the structure. The temperature of the inner surface of the panel reaches an acceptable maximum $T_{w,\text{max}}$ half an hour after the end of the intense heating (Fig. 4(b)).

### 3.2. Optimal mechanical performance

We consider the following optimal design problem: Find the geometry of the panel with minimal mass and satisfying structural strength constraints:

$$m(t_f, t_e, h, d_f, N) \rightarrow \text{min},$$

$$K_y > 1, K_{y,f} > 1, K_y > 1, K_{y,f} > 1, K_y > 1, K_{y,f} > 1, K_y > 1, K_{y,f} > 1, K_y > 1$$

(20)

The solution of this problem implies that the panel with optimal mechanical properties has the smallest overall thickness of 22 mm and almost rectangular web (see Fig. 3b and Table 3). The most significant restriction in the search for the solution is the stability condition of face sheets under compression $K_y > 1$. It is seen from Table 3 that four other geometric parameters can be fixed in the search. Face and web thickness, core depth and number of corrugation pitches over the panel length could have minimum allowable values and only the distance between the web elements should be selected in such a way that the faces and web elements are rather narrow and do not lose stability.

The applied strength and stability conditions in problem (20) are quite common and have been well-tested [1,3–5,7,21]. The condition of stability of web elements under impact (17) is non-trivial and requires numerical verification.

The FE simulation of the impact of the panel was done in the Ansys

![Fig. 4. Results of FE modeling of the process of heating and subsequent cooling of panel for a minimum water discharge density $\eta_{\text{min}}$ in the cooling system, (a) the distribution of panel temperature across the thickness at different time moments, (b) the variation of the maximal and minimal temperature of the inner surface of the panel with time; the temperature distribution in unit cell is shown at the time of maximum heating of the bottom surface.](image-url)
system, in the module for transient analysis using finite strain formulation. A fragment of the panel with optimal mechanical performance was considered; it had length of 100 mm and width of one core wave, that, in the case of an optimum geometry, is 120 mm (Fig. 5a). To be able to simulate the effect of buckling, web elements in the panel were built with given small curvature. The radius of their curvature is 10 m while its height is about 2 cm. Such geometry, with “almost straight web elements”, allows modeling the effects of stability loss in nonlinear transient calculations. At edges of the panel, periodic boundary conditions were set. The initial velocity of the entire model was determined in the vertical direction. On the lower surface of the panel, the distributed mass related to the given dimensions of the fragment was set. On the top surface of the panel, the condition of contact with a flat obstacle was used by setting zero vertical displacements. The problem was solved for the first 2 s of the impact, without damping.

The calculations determine that the deformed state of the panel. It has been found that, at low impact velocities, the web walls retain their shape (Fig. 5b); however, if the impact velocity exceeds certain critical value \(v_{0_{cr}}\), the deformation pattern of the walls changes – the buckling and bending are observed (Fig. 5c). Thus, the 3D model predicts buckling of web elements under impact. We also varied the thickness of web elements and determined the speeds \(v_{0_{cr}}\) that lead to the web elements buckling.

The dependence of the critical impact velocity \(v_{0_{cr}}\) on the thickness of web elements \(t_c\) is shown in Fig. 6 where points show the results of numerical simulation, and lines show the analytical predictions obtained from condition (17). If in this condition we assume the equality, take \(n = 32\), express \(v_0\) in \(t_c\), and take the values of all other parameters from the initial data and solution of the problem (20) (see Table 3), we obtain \(v_0 \approx 7 \sqrt{n t_c}\). This dependence is shown in Fig. 6 by lines, for different values of \(n\). It is seen that the analytical solution agrees well with numerical calculations for \(n = 32\). In this case, the criterion used in the design \(K_e > 1\) can be considered as reliable. We note that due to the presence of damping in the real construction, the critical impact velocities will be higher and in fact an additional strength margin will be realized.

3.3. Optimal combined thermo-structural design

We now consider the optimization problem taking into account all of the constrains for thermal protection and strength, and also taking into account the possibility of variation of the water discharge density in the external cooling system:

\[
\begin{align*}
\min & \quad m(t_f, t_e, h_c, d_f, N) \\
\text{subject to} & \quad \eta_{\text{min}} \leq \eta \leq \eta_{\text{max}}, \quad K_{T_{\text{min}}} > 0, \quad K_{T_{\text{max}}} > 0, \\
& \quad K_f > 1, \quad K_{y_f} > 1, \quad K_{y_d} > 1, \quad K_{y_T} > 1, \quad K_{y_g} > 1, \quad K_{y_{buck}} > 1, \quad K_c > 1, \quad K_{T_{buck}} > 1
\end{align*}
\]  

(21)

The solution of problem (21) is the geometry of the panel shown in Fig. 3c. Parameters of the panel optimal geometry are presented in Table 3. The unit cell of the panel has sufficiently large total thickness to satisfy the requirements of thermal protection. Due to this, it is necessary to increase the web thickness, to prevent its buckling under mechanical loading and under non-uniform heating. This leads to an increase of the panel mass.

Essential constraints in problem (21) are the thermal insulation condition at low temperatures \(K_{T_{\text{min}}} > 0\), the condition of faces stability under compression \(K_{y_f} > 1\) and conditions of web stability under compression \(K_{y_d} > 1\) and under non-uniform heating \(K_{T_{buck}} > 1\). Note

Fig. 5. Transient simulations of the panels fragment under impact, a: loading scheme, b: stability of the web elements under low speed of the impact, c: buckling of the web elements under high speed of the impact. The color in figures b and c shows the magnitude of the total displacements. Real scale of deformations is used.

Fig. 6. Dependence of the impact critical velocity \(v_{0_{cr}}\) that induced the web elements buckling on web thickness \(t_c\). Analytical solution is shown by lines, FE simulation – by points.
that the last condition can be satisfied only by lowering the temperature of the panel heating. Hence it is necessary to set an increased discharge density in the cooling system \( \eta > \eta_{\text{min}} \). The not essential parameters of the problem (21) are the faces thickness and the number of web pitches over panel length. The solution has monotone dependence on them, thus we assign them the minimum values in the predefined ranges (see Table 3).

Based on the obtained results shown in Fig. 7, the failure/thermal protection map is constructed for the variation of core depth and web thickness and for different discharge densities in the cooling system. It is seen that, if the minimal discharge density is used, the problem (21) has no solution. In this case, the web thermal stability area of parameters values do not overlap with the areas of other criteria satisfaction (see red dotted line in Fig. 7). As the discharge in the cooling system increases, the panel temperature decreases and the web thermal buckling criterion shifts higher (black dashed line in Fig. 7). Thus, a region of optimal values can be identified (shaded area in Fig. 7). The minimum mass of the panel is realized for the minimum allowable values of the parameters found from the solution of the optimization problem. This solution is shown by the point in Fig. 7.

The found optimal variant of the geometry (Fig. 3c) must be verified in FE simulation. Of all the restrictions we used, the condition for the thermal buckling of the web elements has not been verified, which in this case turned out to be the most important one in the problem (21). To test it, we perform FE modeling of thermomechanical behavior of the panel under non-uniform heating. As shown in [11], the web thermal buckling in the corrugated core sandwich panel occurs at maximal temperature drop between its outer and inner surfaces. In the considered problem, the maximal temperature difference (\( -20/90 \^\circ\text{C} \)) occurs during the first eight minutes of heating. The maximal average panel temperature is realized at the end of this period.

Thus, in the FE thermomechanical simulations we specify the temperature field realized in the panel after 8 min of heating. This temperature distribution, found from the 2-D heat conduction problem, was prescribed in the 3-D model (Fig. 8a). Buckling analysis was carried out based on the static thermoelasticity solution in the Ansys system. At the edges of the unit cell in the direction of corrugation, the periodic boundary conditions were set. It has been found that for the optimal panel geometry (Fig. 3c) the buckling safety factor is 2.34 and the first buckling mode refer specifically to the web elements (Fig. 8b).

4. Conclusions

Optimal design solutions for the corrugated core sandwich thermal barrier panel are presented. It is shown that thermal protection under the conditions of burning oil field can be provided using an active external cooling system. However, it is not enough to use the minimal discharge density in the cooling system, since, in addition to thermal protection, this system must ensure the preservation of the load bearing capacity of the structure. Using increased discharge ensures stability of the web elements under non-uniform heating inside the panel.

As found from the thermo-structural optimization, the panel geometry (Fig. 3c) is not the intermediate one, between the optimal variants found for the best mechanical or thermal performance separately (Fig. 3a, b). In the combined thermo-structural problem, it is necessary to increase the total thickness of the panel to meet the requirements of the thermal protection, which leads to increase the thickness of the web elements to prevent buckling under compression, impact or non-uniform heating of the panel. As web thickness increases, the equivalent thermal conductivity of the panel also increases, and this leads to the need for an additional increase of its total thickness. These iterations converge, but the optimal geometry of the panel has a greater total thickness and a larger mass, than the panels designed only with constraints of mechanical strength or heat protection.

To further improve the considered structure, it is necessary to solve the problem of stability of face and web elements under mechanical and thermal loads. These are the main criteria that limit the strength of the panel (see Table 3). Solution of this problem is possible, e.g. with the use of foam insulation instead of fibrous one inside the corrugated core of the panel. The foam insulation has relatively high stiffness, and it must prevent the stability loss of the panel elements, see [24], [25]. However, the thermal conductivity and density of the foams are higher than the thermal conductivity of fibrous materials, so the efficiency of the foam-filled corrugated core sandwich panels should also be checked in the context of the corresponding optimization problem.
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